Solution to Problem Set 12
Optical Waveguides and Fibers (OWF)

Problem 1: Multi-mode interference (MMI) coupler

Consider a multi-mode interference (MMI) coupler having two input ports and two output ports, see Fig. 1. The device should operate at a wavelength of 1550 nm. Assume that the waveguide core of the MMI coupler is made of silicon ($n_1 = 3.48$) having a thickness $h = 220\,\mathrm{nm}$, and that it is surrounded by silicon dioxide ($n_2 = 1.44$). The width of the MMI section is fixed to $w = 5\,\mu\mathrm{m}$, and we need to adjust its length L, such that the power from a single input port is divided equally to the two output ports.

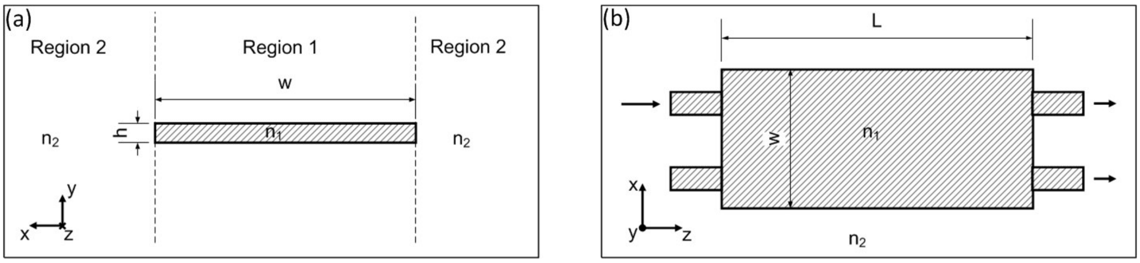

Figure 1: Multi-mode interference coupler: Definition of the geometry, front view (a) and top view (b) of the structure.

a) Reduce the three-dimensional structure to a two-dimensional problem in the x-z plane by applying the effective-index method to the regions 1 and 2. Calculate the corresponding effective indexes n_{1e} and n_{2e} for the fundamental TE slab modes (electric field parallel to the x- axis) using the Matlab code that you wrote for solving Problem Set 4, or the online solver available on the website http://www.computational-photonics.eu/oms.html. Note that the structure has a high index contrast. Nevertheless, the effective index method provides here a good estimation and a starting point for further numerical optimization.

Solution:

Region 2 is homogeneous with $n_{2e} = n_2 = 1.44$. For region 1 we can calculate the TE slab-mode with $h = 0.22\,\mathrm{\mu m}$. This results in an effective refractive index of $n_{1e} = 2.814$ for region 1.

b) Calculate the beat length which is defined by

$$L_\pi = \frac{4 n_{1e} w^2}{3\lambda}, \tag{1}$$

where n_{1e} represents the effective index of the fundamental TE mode of the 220 nm thick silicon slab waveguide in region 1. How long should the 2×2 MMI coupler be?

Solution:

From Fig. 6.4 in the lecture notes one can read that for a coupler length of $\frac{3}{2}L_\pi$ the input power of one input port is distributed equally in both output ports. The MMI length can then be calculated with:

$$L_\pi = \frac{4 n_{1e} w^2}{3\lambda} = 61.32\,\mathrm{\mu m},$$
$$L_{3dB} = \frac{3}{2} L_\pi = \frac{3}{2}\frac{4 n_{1e} w^2}{3\lambda} = 91.98\,\mathrm{\mu m}.$$

c) Which assumptions have been made in deriving the beat-length in Eq. (1)? Why does n_{2e} not play any role?

Solution:

The first assumption made is that a weakly guiding waveguide is considered, which also incorporates the assumption of negligible inhomogenities and therefore enables the description of the modes via the scalar wave equation.

The second assumption made is that only modes far from cut-off are considered, leading to the approximation $u_m = \frac{w}{2}\sqrt{n_{1e}^2 k_0^2 - \beta^2} \approx (m+1)\frac{\pi}{2}$. For large $V = \frac{w}{2}\frac{2\pi}{\lambda}\sqrt{n_1^2 - n_2^2}$, this approximation is valid, i.e., the wavguide thickness is large compared to the wavelength. Figure 2 shows solutions for the eigenvalue equation of a waveguide, where intersections of the curves yield solutions for u_m. As it can be seen, these approach multiples of $\frac{\pi}{2}$ for large values of V. Such modes do not extend into the cladding material, which is why $n_{2,e}$ does not play a role.

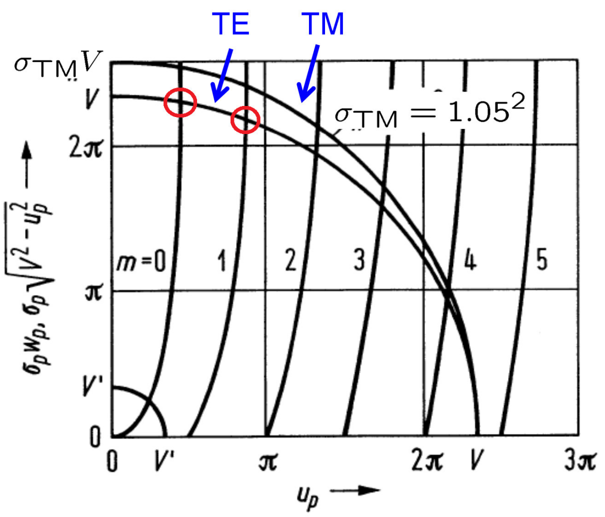

Figure 2: Solutions for eigenvalue equation of a waveguide. Red marked are solutions, whose corresponding values of u_m are close to multiples of $\frac{\pi}{2}$.

d) Apply again the effective-index method to the slab waveguide defined by n_{1e} and n_{2e} for calculating the quantity $\beta_0 - \beta_1$. Compare the result with the value given by the equation

$$\beta_0 - \beta_m = \frac{m(m+2)\pi}{3L_\pi}, \tag{2}$$

which has been assumed in deriving the imaging properties of the MMI based on the beat-length L_π.

Solution:

The structure defined by region 1 and region 2 can be used to calculate the TM modes with the effective index method. This section now is highly multimode, and we get for the first two modes:

$$\text{TM0: } \beta = 11.5420\frac{1}{\mu\text{m}}, \quad n_e = 2.8473,$$
$$\text{TM1: } \beta = 11.4913\frac{1}{\mu\text{m}}, \quad n_e = 2.8348.$$

We can use this to calculate $\beta_0 - \beta_1 = 0.0507\frac{1}{\mu\text{m}}$. Using Eq. 2 and the result from b) we can write $\frac{m(m+2)\pi}{3L_\pi} = \frac{3\pi}{3L_\pi} = 0.0512\frac{1}{\mu\text{m}}$. As it can be seen, the results of the effective index method and the consideration of modes far from cut-off yield similar results, differing by approximately 1%.

e) Which value of w should you insert in Eq. (2) in order to get a perfect agreement with the value of $\beta_0 - \beta_1$ calculated by means of the effective-index method?

Solution:

Using Eq. (1) and Eq. (2) with $\beta_0 - \beta_1 = \frac{\pi}{L_\pi}$, we can calculate the width:

$$w = \sqrt{\frac{3\pi\lambda}{4n_{1e}(\beta_0 - \beta_1)}} = 5.03\mu\text{m}.$$

Supplementary information about the length of a 3dB-coupler

The length of a 3dB-coupler is given by:

$$L_{3dB} = \frac{3}{2}L_\pi.$$

In the following section, it is going to be shown why after this propagation distance the power of an incoming signal is split equally into two parts. For a propagation into z-direction and an orientation of the ports of a MMI-coupler along x-direction the field inside can be written as

$$\phi(x,y,z) = \sum_m a_m \Psi_m(x,y) \exp\left(j\frac{m(m+2)\pi}{3L_\pi}z\right) \exp(j\beta_0 z),$$

where a_m are the mode coefficients, $\Psi_m(x,y)$ are the field distributions in x- and y-direction and β_0 is the propagation constant of the fundamental mode. Depending on the symmetry of the mode, the term $\exp\left(j\frac{m(m+2)\pi}{3L_\pi}\frac{3}{2}L_\pi\right)$ gives different results:

$$\begin{aligned} m \text{ even}: \quad & \exp\left(j\tfrac{m(m+2)\pi}{3L_\pi}\tfrac{3}{2}L_\pi\right) = 1, \quad \Psi_m(x,y) = \Psi_m(-x,y), \\ m \text{ odd}: \quad & \exp\left(j\tfrac{m(m+2)\pi}{3L_\pi}\tfrac{3}{2}L_\pi\right) = j, \quad -\Psi_m(x,y) = \Psi_m(-x,y). \end{aligned}$$

For the even modes, $\Psi_m(x,y)\cdot 1$ can be written as follows:

$$\begin{aligned} \Psi_m(x,y)\cdot 1 &= \frac{1-j}{2}\Psi_m(x,y) + \frac{1+j}{2}\Psi_m(x,y) \\ &= \frac{1-j}{2}\Psi_m(-x,y) + \frac{1+j}{2}\Psi_m(x,y). \end{aligned}$$

For the odd modes, $\Psi_m(x,y)\cdot j$, it holds:

$$\begin{aligned} \Psi_m(x,y)\cdot j &= j\left(\frac{1+j}{2}\Psi_m(x,y) + \frac{1-j}{2}\Psi_m(x,y)\right) \\ &= \frac{j-1}{2}\Psi_m(x,y) + \frac{j+1}{2}\Psi_m(x,y) \\ &= -\frac{1-j}{2}\Psi_m(x,y) + \frac{j+1}{2}\Psi_m(x,y) \\ &= \frac{1-j}{2}\Psi_m(-x,y) + \frac{j+1}{2}\Psi_m(x,y). \end{aligned}$$

So in fact, the even and the odd modes have the same prefactors and the whole field $\phi(x,y,\frac{3}{2}L_\pi)$ can be written as:

$$\begin{aligned} \phi(x,y,\frac{3}{2}L_\pi) &= \sum_m a_m\left(\frac{1-j}{2}\Psi_m(-x,y) + \frac{j+1}{2}\Psi_m(x,y)\right)\exp\left(j\beta_0\frac{3}{2}L_\pi\right) \\ &= \left(\frac{1-j}{2}\phi(-x,y,0) + \frac{j+1}{2}\phi(x,y,0)\right)\exp\left(j\beta_0\frac{3}{2}L_\pi\right). \end{aligned}$$

As it can be seen from the last result, the initial field at a position x' is split equally into the positions x' and $-x'$, where further waveguides are located to couple light in and out of the MMI-coupler.

Questions and Comments:

Aleksandar Nesic
Building: 30.10, Room: 2.32-2
Phone: 0721/608-42480
aleksandar.nesic@kit.edu

Philipp Trocha
Building: 30.10, Room: 2.32-2
Phone: 0721/608-42480
philipp.trocha@kit.edu